

Ex 1.6

$$(9) \quad f(g(x)) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$$

we obtain

$$f(x) = \frac{x+1}{x-1} \quad \text{and} \quad g(x) = \sqrt{x}$$

Decomposition is not unique!

$$\text{Let's rewrite } f(g(x)) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$$

$$f(g(x)) = \frac{\sqrt{x}-1+1+1}{\sqrt{x}-1}$$

$$\Rightarrow f(g(x)) = \frac{\sqrt{x}-1+2}{\sqrt{x}-1}$$

$$\text{Let } g(x) = \sqrt{x}-1$$

so we have

$$f(g(x)) = \frac{g(x)+2}{g(x)}$$

$$\Rightarrow f(x) = \frac{x+2}{x} \quad \therefore f(x) = \frac{x+2}{x} \quad \text{and} \quad g(x) = \sqrt{x}-1$$

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Revenue Product= Revenue that depends on
the no. of employees/workers

This is true in service sectors.

It is also true in production sectors.

Revenue depends on the quantity
produced and sold.And quantity depends on the no. of
workers you hire.

In summary,

Revenue depends on quantity, $R(q)$,and quantity depends on the no. of workers, $q(w)$

So Revenue product is

$$R(q(w)) = R(w)$$

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(13) A local Sashidharam construction

Solution:-

$$\text{Given } q(w) = \sqrt{w} \quad ; \quad 0 \leq w \leq 100$$

$$\text{and } R(q) = 300q^2 + 50q$$

$$\text{so } R(q(w)) = 300(\sqrt{w})^2 + 50\sqrt{w} \quad \leftarrow \text{since } q = \sqrt{w}$$

$$\Rightarrow R(w) = 300w + 50\sqrt{w} \quad \left(\sqrt{w} \right)^2 = \begin{cases} w, & \text{if } w \geq 0 \\ -w, & \text{if } w < 0 \end{cases}$$

What is R when $w = 100$

$$\text{so } R(100) = 300(100) + 50\sqrt{100}$$

$$\Rightarrow R(100) = 30000 + 500 \\ = \$30,500$$

Definition of $(\sqrt{w})^2$

$$(\sqrt{w})^2 = \begin{cases} w, & \text{if } w \geq 0 \\ -w, & \text{if } w < 0 \end{cases}$$

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1.7 Inverse Functions

In a regular function, $y=f(x)$, we assign a value to x and obtain a value for y .

The inverse is asking that given a value of y , what is the value of x that returns this value of y ?

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Example

① I want to buy 3 apples. How much will it cost?

$$x = 3 \text{ apples, } y = \$???$$

② Inversely, I want to buy \$5 worth of apples. How apples can I get?

$$y = \$5, \quad x = ??? \text{ apples}$$

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Rule Only one-to-one functions have inverses.

Function:-

Every x corresponds to only one y .

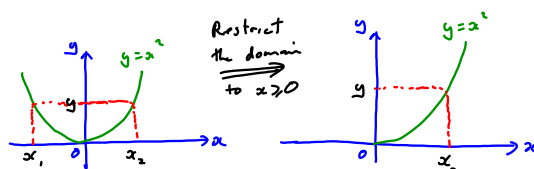
But a y may correspond to more than one x .

One-to-one Function:-

Every x corresponds to only one y .

And every y corresponds to only one x .

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Example Regular function $y=x^2$ 

Note:- we can force a non one-to-one function to become one-to-one by restricting its domain.

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Once we get a one-to-one function, we can find its inverse using the following steps:-

① Let $y=f(x)$

② Rewrite the function as x in terms of y .

③ Replace x with y , and y with x .

$\therefore$ The inverse is written as

$$f^{-1}(x) \quad f^{-1}(x) \text{ and } f(x) \text{ are inverses of each other.}$$

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Ex 1.7

④ Find the inverse of

$$y = \frac{2x+3}{4-x}$$

$$\Rightarrow y(4-x) = 2x+3 \quad \leftarrow \text{cross-multiply}$$

$$\Rightarrow 4y - xy = 2x+3 \quad \leftarrow \text{distribute } y \text{ into } 4-x$$

$$\Rightarrow -xy - 2x = 3-4y \quad \leftarrow \text{Collect all terms with } x \text{ to one side and all terms without } x \text{ to the other}$$

$$\Rightarrow -x(y+2) = 3-4y \quad \leftarrow \text{Factor out } -x \text{ at the Left side}$$

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$$\Rightarrow x = \frac{3-4y}{-(y+2)} = \frac{4y-3}{y+2} \quad \leftarrow \text{Now we have } x \text{ in terms of } y$$

So the inverse is

$$y^{-1}(x) = \frac{4x-3}{x+2} \quad \leftarrow \text{Replace } y \text{ with } x \text{ and } x \text{ with } y$$

$\therefore$ The inverse of $y(x) = \frac{2x+3}{4-x}$ is $y^{-1}(x) = \frac{4x-3}{x+2}$

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Important :-

$$y^{-1}(x) \neq \frac{1}{y(x)}$$

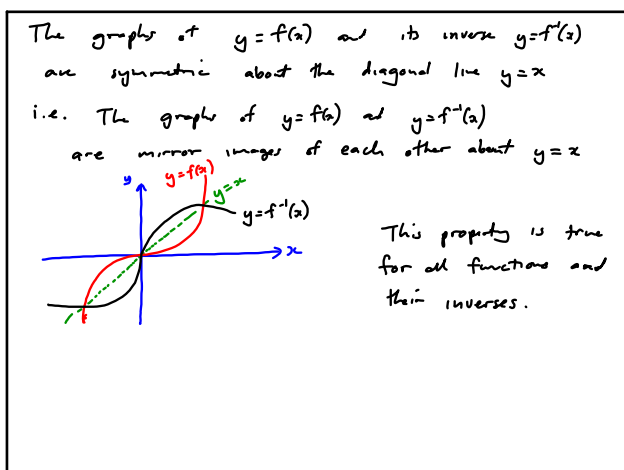
Similarly,

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

Numbers :-

$$x^{-1} = \frac{1}{x} \quad \checkmark$$

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Some things that may interest you

① If $f(x) = kx^p$

Let $y = kx^p \quad \leftarrow \text{Let } y=f(x)$

$$\Rightarrow x^p = \frac{y}{k}$$

$$\Rightarrow x = \sqrt[p]{\frac{y}{k}} = \left(\frac{y}{k}\right)^{\frac{1}{p}} \quad \leftarrow \text{Rewrite } x \text{ in terms of } y$$

$\therefore f^{-1}(x) = \sqrt[p]{\frac{x}{k}} = \left(\frac{x}{k}\right)^{\frac{1}{p}} \quad \leftarrow \text{Replace } y \text{ with } x.$

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② $f(f^{-1}(x)) = x$
and $f^{-1}(f(x)) = x$

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③ $p(x) = 4-3x$ and $r(x) = \frac{4-x}{3}$

(i) $p(r(x)) = 4-3r$

$$= 4-3\left(\frac{4-x}{3}\right)$$

$$= 4-(4-x)$$

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$$\Rightarrow p(r(x)) = 4-4+x$$

$$\Rightarrow p(r(x)) = x$$

$\therefore p(x)$ and $r(x)$ are inverses of each other.

OR

$$r(p(x)) = \frac{4-p}{3}$$

$$= \frac{4-(4-3x)}{3} = \frac{4-4+3x}{3}$$

$$\Rightarrow r(p(x)) = \frac{3x}{3}$$

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$$\Rightarrow r(p(x)) = x$$

$\therefore p(x)$ and $r(x)$ are inverses of each other.

Note:-

If $p(r(x)) \neq x$ and $r(p(x)) \neq x$

then $p(x)$ and $r(x)$ are not inverses of each other.

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